Extension of Marcatili's Analytical Approach for 220 nm high waveguides in SOI technology

W.J. Westerveld[1,2], S.M. Leinders[1], J. Pozo[2], K.W.A. van Dongen[1], M. Yousefi[3], and H. P. Urbach[1]

[1] Department of Imaging Science & Technology, Delft University of Technology,
Van der Waalsweg 8, 2628CH Delft, The Netherlands
[2] TNO, Stieltjesweg 1, 2628 CK Delft, The Netherlands
[3] Photonic Sensing Solutions, Haarlemmerstraat 141, 1013EN Amsterdam, The Netherlands

Marcatili's famous approximate analytical description of light propagating through rectangular dielectric waveguides, published in 1969, gives accurate results for low-index-contrast waveguides. However, silicon-on-insulator (SOI) technology has become one of the focus platforms for integrated optics over the last decade, and the silicon waveguides have a very high-index-contrast. In this paper, we improve Marcatili's model by adjusting the amplitudes of the components of the electromagnetic fields. Our improved method shows much better agreement with rigorous numerical simulations of SOI waveguides.

Introduction

Silicon photonic integrated circuits (ICs) have gained large interest over the last decade. Several microelectronic research institutes have tailored CMOS processes to the demands of photonic ICs, and now offer wafer-scale fabrication [1]. These circuits usually have 220 nm high silicon ridge waveguides on top of a SiO_2 isolating layer. The height of these guides is solely defined by the SOI layer stack providing higher fabrication yield, and these guides only support TE-like modes which reduces complexity. We recently extended Marcatili's approximate analytical approach for the description of light propagation in rectangular waveguides to the regime of (silicon) high-index-contrast waveguides [2, 3], and hereby apply this method to the 220 nm high SOI guides. After a general description of electromagnetic waves in dielectric guides, we make an *Ansatz* on the form of these modes, and then lay down the boundary conditions that should be fulfilled. Then we propose an approximate solution, which is verified with rigorous numerical computation.

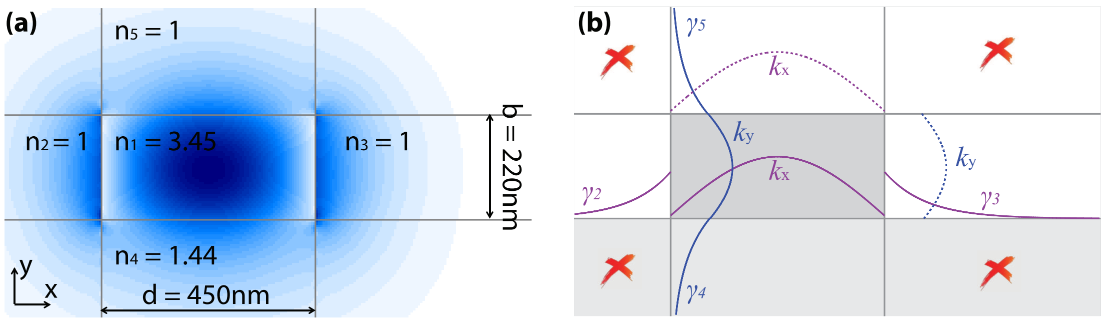

Figure 1: Cross-section of a SOI waveguide. (a) Waveguide definition with E_x component of the fundamental mode in color. (b) Outline of analytical approximate method.

Electromagnetic modes in silicon dielectric waveguides

Fig. 1 depicts a typical SOI waveguide, whose core has a higher refractive index (n_1) than its surroundings ($n_2 - n_4$). We consider a monochromatic guided mode propagating in the waveguide direction (z) with angular frequency ω, propagation constant β, and electric field profile $\boldsymbol{E}(x,y)$:

$$\boldsymbol{\mathcal{E}}(x,y,z,t) = \mathrm{Re}\left\{\boldsymbol{E}(x,y)\exp[\imath(\omega t - \beta z)]\right\}. \tag{1}$$

The refractive index invariant in the z-direction, allowing the full electromagnetic field to be expressed in its longitudinal components only [4]:

$$E_x = \frac{-\imath}{K_j^2}\left(\beta\frac{\partial E_z}{\partial x} + \omega\mu_0\frac{\partial H_z}{\partial y}\right), \quad H_x = \frac{-\imath}{K_j^2}\left(\beta\frac{\partial H_z}{\partial x} - \omega\varepsilon_0 n_j^2\frac{\partial E_z}{\partial y}\right), \tag{2}$$

$$E_y = \frac{-\imath}{K_j^2}\left(\beta\frac{\partial E_z}{\partial y} - \omega\mu_0\frac{\partial H_z}{\partial x}\right), \quad H_y = \frac{-\imath}{K_j^2}\left(\beta\frac{\partial H_z}{\partial y} + \omega\varepsilon_0 n_j^2\frac{\partial E_z}{\partial x}\right), \tag{3}$$

where μ_0 and ε_0 are the permeability and the permittivity of vacuum, and $K_j^2 \equiv n_j^2 k_0^2 - \beta^2$, with free-space propagation constant $k_0 = \omega/c$, and c the speed of light in vacuum. All components satisfy the reduced wave equation, thus [4]

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + K_j^2 E_z = 0. \tag{4}$$

Ansatz on the shape of the field

We assume that the modal field has most of its energy in the center of the waveguide (such as in Fig. 1a), and also that the mode is “TE-like” with the majority of the electromagnetic energy in the E_x and H_y components. From this we make the *Ansatz* that $E_x(x,y)$, and $H_y(x,y)$, are proportional to $\cos[k_x(x+\xi)]\cos[k_y(y+\eta)]$, with maximum field in the center. Outside the core, the fields decay exponentially, while the transversal profile of the field is identical to that in the core (Figs. 1b and 2a). The outer quadrants are neglected with the motivation that the fields are small in these regions. Fig. 2b presents the *Ansatz* on the full field, expressed in its longitudinal components. We choose to obey Maxwell's equations in all regions 1-5, and express β, and γ_2 - γ_5 in terms of k_x and k_y employing the wave equation (4)

$$\beta = \sqrt{n_1^2 k_0^2 - k_x^2 - k_y^2}, \tag{5}$$

$$\gamma_2^2 = (n_1^2 - n_2^2)k_0^2 - k_x^2, \quad \gamma_3^2 = (n_1^2 - n_3^2)k_0^2 - k_x^2, \tag{6}$$

$$\gamma_4^2 = (n_1^2 - n_4^2)k_0^2 - k_y^2, \quad \gamma_5^2 = (n_1^2 - n_5^2)k_0^2 - k_y^2. \tag{7}$$

All errors of the approximation manifest themselves at the interfaces of the core of the waveguide. Field amplitude A_1 is employed to normalize the mode to a power flux of unity, such that the description has A_2 - A_{10}, ξ, η, k_x and k_y, as free parameters (13 in total).

(a)	region 5: $E_x \propto \cos[k_x(x+\xi)]\exp[-\gamma_5(y-b/2)]$	
region 2: $E_x \propto \exp[\gamma_2(x+d/2)]\cos[k_y(y+\eta)]$	region 1: $E_x \propto \cos[k_x(x+\xi)]\cos[k_y(y+\eta)]$	region 3: $E_x \propto \exp[-\gamma_3(x-d/2)]\cos[k_y(y+\eta)]$
	region 4: $E_x \propto \cos[k_x(x+\xi)]\exp[\gamma_4(y+b/2)]$	

(b)	$E_z = A_9\sin[k_x(x+\xi)]\exp[-\gamma_5(y-b/2)]$ $H_z = A_{10}\cos[k_x(x+\xi)]\exp[-\gamma_5(y-b/2)]$	
$E_z = A_3\exp[\gamma_2(x+d/2)]\cos[k_y(y+\eta)]$ $H_z = A_4\exp[\gamma_2(x+d/2)]\sin[k_y(y+\eta)]$	$E_z = A_1\sin[k_x(x+\xi)]\cos[k_y(y+\eta)]$ $H_z = A_2\cos[k_x(x+\xi)]\sin[k_y(y+\eta)]$	$E_z = A_5\exp[-\gamma_3(x-d/2)]\cos[k_y(y+\eta)]$ $H_z = A_6\exp[-\gamma_3(x-d/2)]\sin[k_y(y+\eta)]$
	$E_z = A_7\sin[k_x(x+\xi)]\exp[\gamma_4(y+b/2)]$ $H_z = A_8\cos[k_x(x+\xi)]\exp[\gamma_4(y+b/2)]$	

Figure 2: (a) Shape of the dominant electromagnetic field components E_x and H_y. (b) *Ansatz* describing the modal electromagnetic field in terms of E_z and H_z.

Boundary conditions

At interfaces of the core of the waveguide, continuity of the electromagnetic field components tangential to these interfaces is required, adding up to 4 $\times$ 4 = 16 electromagnetic boundary conditions. With these conditions satisfied, the normal components automatically obey Maxwell's equations. We solve for k_y, η, and A_2 from boundary conditions on the horizontal interfaces

$$\tan[k_y b] = k_y(\gamma_4+\gamma_5)/(k_y^2-\gamma_4\gamma_5), \qquad A_2 = A_1\beta k_y/(\omega\mu_0 k_x), \tag{8}$$

while we find k_x, ξ, and also A_2 from the conditions on the vertical interfaces

$$\tan[k_x d] = n_1^2 k_x(n_3^2\gamma_2+n_2^2\gamma_3)/(n_2^2n_3^2k_x^2-n_1^4\gamma_2\gamma_3), \qquad A_2 = A_1\omega\varepsilon_0 n_1^2 k_y/(\beta k_x). \tag{9}$$

It can be seen that the horizontal and the vertical interfaces require a different ratio A_2/A_1, i.e. a different H_z/E_z in the core. Thus the *Ansatz* has no solutions that exactly obey the boundary conditions at all interfaces simultaneously. In what follows, the 13 free parameters are chosen such that the error in the 16 boundary conditions is minimal.

Approximate aproaches

We argue that the dominant boundary conditions for determining k_y and η are at the horizontal interfaces, while the vertical interfaces dominate k_x and ξ, such that we can compute k_x, k_y and β from Eqs. (8), (9) and (5). In Fig. 3 we compare the analytical and numerical data and find that the effective index, $n_{\text{eff}} = \beta/k_0$, is accurately found within 1%.

Marcatili has developed a widely used analytical approach for low-index-contrast waveguides [2]. For propagating modes in these guides, $k_0 n_j \approx \beta$ because modes are not guided otherwise, so $k_x, k_y << k_0 n_j$. Choosing $E_y = 0$ gives a modal field profile that is continuous on the horizontal interfaces, while it obeys the conditions on the vertical interfaces when neglecting terms on the order of $(k_x/k_0 n_j)^2$. However, these terms are sometimes even larger than unity for high-index-contrast guides.

We propose the *Extended* $E_y \approx 0$ *method* in which the fields are also continuous at the horizontal interfaces. At the vertical interfaces, E_x and H_z are matched, while E_z and H_x are not. E_x is a dominant component, H_z is chosen over E_z because an infinitely

wide (slab) waveguide has $E_z = 0$, and $H_x \propto \sin[k_x(x+\xi)]\sin[k_y(y+\eta)]$ is a weak field component with high intensity at the corners of the guide.
We also propose the *amplitude optimization method* in which the energy density associated with the discontinuity of the tangential field across the boundaries, U_{mm}, is minimized.

$$U_{\mathrm{mm}} \equiv \frac{\varepsilon_0}{4l}\oint (n^+ + n^-)^2 \cdot \left|\hat{\mathbf{v}} \times \left(\boldsymbol{E}^+ - \boldsymbol{E}^-\right)\right|^2 dl + \frac{\mu_0}{l}\oint \left|\hat{\mathbf{v}} \times \left(\boldsymbol{H}^+ - \boldsymbol{H}^-\right)\right|^2 dl. \qquad (10)$$

The line integral runs along the circumference, $l = 2(b+d)$, of the waveguide in the (x,y)-plane. $(\boldsymbol{E}^+ - \boldsymbol{E}^-)$ is the discontinuity of the field just outside (+) and inside (-) the core. $\hat{\mathbf{v}}$ is a unit vector orthogonal to the surface used to select the tangential components via the cross product. At the interface, an average refractive index $(n^+ + n^-)/2$ is assumed.

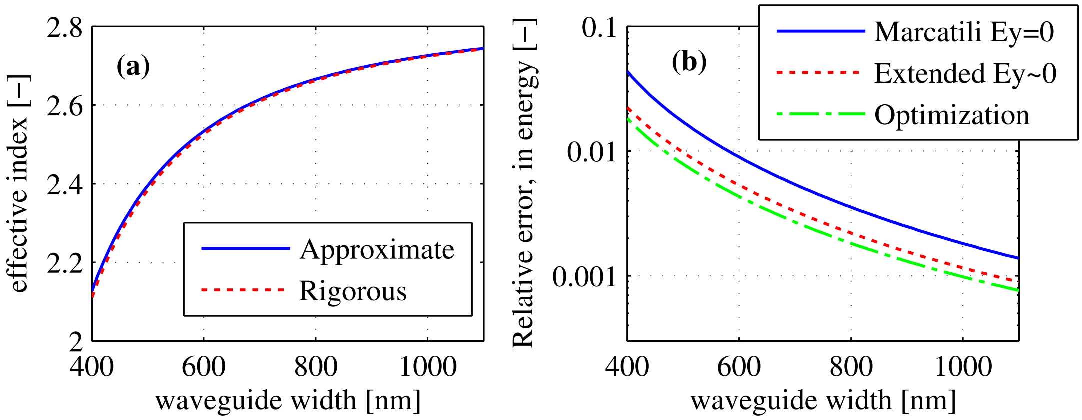

Figure 3: Results compared with rigorous mode solver employing the film mode matching method (FimmWave, Photon Design, Oxford, UK). (a) Effective index. (b) Energy in the difference field between the approximate methods and the rigorously computed field, normalized to the energy in the rigorously computed field.

Conclusion

An alternative derivation for Marcatili's method was presented, together with two improvements on the modal field. For 400 nm wide SOI waveguides, the error in effective index is below 1%. The modal fields from Marcatili $E_y = 0$, *Extended* $E_y \approx 0$ and *amplitude optimization* methods show a difference with rigorous calculations of 4.4%, 2.2%, and 1.8% (in energy), respectively. Marcatili's method is widely used in education, and frequently occurs in textbooks. With our work, the development of intuitive analytical methods now follows the technological developments.

References

[1] J. Pozo *et al.*, "ESSenTIAL: EPIXfab services specifically targeting (SME) industrial takeup of advanced silicon photonics," 14th International Conference on Transparent Optical Networks (ICTON) 2012 , Coventry, England, July 2012

[2] E. Marcatili, "Dielectric rectangular waveguide and directional coupler for integrated optics," *The Bell System Technical Journal*, vol. 48, pp. 2071–2121, Mar. 1969.

[3] W.J. Westerveld *et al.*, "Extension of Marcatili's analytical approach for rectangular silicon optical waveguides," *Journal of Lightwave Technology*, vol. 30, pp. 2388–2401, June 2012.

[4] D. Marcuse, *Theory of dielectric optical waveguides*. San Diego: Academic Press Inc, 1991.